Homework Assignment # 3 (Due April 4, 2017)

Chapter 5

1) Beginning with Equation (5.5), use Equation (5.6) to eliminate $V_c$ and $V_d$ to arrive at the result

$$w_{cycle} = nR \left( T_{hot} - T_{cold} \right) \ln \frac{V_b}{V_a}.$$ 

2) Consider the reversible Carnot cycle shown in Figure 5.2 with 1 mol of an ideal gas with $C_{V,m} = \frac{3}{2}R$ as the working substance. The initial isothermal expansion occurs at the hot reservoir temperature of $T_{hot} = 600 \, ^\circ C$ from an initial volume of 3.50 L ($V_a$) to a volume of 10.0 L ($V_b$). The system then undergoes an adiabatic expansion until the temperature falls to $T_{cold} = 150 \, ^\circ C$. The system then undergoes an isothermal compression and a subsequent adiabatic compression until the initial state described by $T_a = 600 \, ^\circ C$ and $V_a = 3.50$ L is reached.

a) Calculate $V_c$ and $V_d$.
b) Calculate $w$ for each step in the cycle and for the total cycle.
c) Calculate $\varepsilon$ and the amount of heat that is extracted from the hot reservoir to do 1.00 kJ of work in the surroundings.
d) Calculate $q$, $\Delta U$, and $\Delta H$ for each step in the cycle and for the total cycle.

3) Calculate $\Delta S$ if the temperature of 1 mol of an ideal gas with $C_{V,m} = \frac{3}{2}R$ is increased from 150 to 350 K under conditions of (a) constant pressure and (b) constant volume.

4) One mole of an ideal gas with $C_{V,m} = \frac{3}{2}R$ undergoes the transformations described in the following list from an initial state described by $T = 300$ K and $P = 1.00$ bar. Calculate $q$, $w$, $\Delta U$, $\Delta H$, and $\Delta S$ for each process.

a. The gas is heated to 450 K at a constant external pressure of 1.00 bar.
b. The gas is heated to 450 K at a constant volume corresponding to the initial volume.
c. The gas undergoes a reversible isothermal expansion at 300 K until the pressure is half of its initial value.

5) One mole of an ideal gas with $C_{V,m} = \frac{5}{2}R$ undergoes the transformations described in the following list from an initial state described by $T = 250$ K and $P = 1.00$ bar. Calculate $q$, $w$, $\Delta U$, $\Delta H$, and $\Delta S$ for each process.
a. The gas undergoes a reversible adiabatic expansion until the final pressure is half its initial value.

b. The gas undergoes an adiabatic expansion against a constant external pressure of 0.500 bar until the final pressure is half its initial value.

c. The gas undergoes an expansion against a constant external pressure of zero bar until the final pressure is equal to half of its initial value.

6) One mole of H₂O(l) is supercooled to -2.25 °C at 1 bar pressure. The freezing temperature of water at this pressure is 0.00 °C. The transformation H₂O(l) → H₂O(s) is suddenly observed to occur. By calculating \( \Delta S \), \( \Delta S_{\text{surroundings}} \), and \( \Delta S_{\text{total}} \), verify that this transformation is spontaneous at -2.25 °C. [Hint: Consider the two pathways at 1 bar: (a) H₂O (l, -2.25 °C) → H₂O (s, -2.25 °C) and (b) H₂O (l, -2.25 °C) → H₂O (l, 0.00 °C) → H₂O (s, 0.00 °C) → H₂O (s, -2.25 °C). Because \( S \) is a state function, \( \Delta S \) must be the same for both pathways.]

7) An ideal gas sample containing 2.50 mol for which \( C_{V,m} = \frac{3}{2} R \) undergoes the following reversible cyclical process from an initial state characterized by \( T = 450 \) K and \( P = 1.00 \) bar:

a. It is expanded reversibly and adiabatically until the volume doubles.

b. It is reversibly heated at constant volume until \( T \) increases to 450 K.

c. The pressure is increased in an isothermal reversible compression until \( P = 1.00 \) bar.

Calculate \( q \), \( w \), \( \Delta U \), \( \Delta H \), \( \Delta S \), \( \Delta S_{\text{surroundings}} \), and \( \Delta S_{\text{total}} \) for each step in the cycle, and for the total cycle. The temperature of the surroundings is 300 K.

8) One mole of a van der Waals gas at 27 °C is expanded isothermally and reversibly from an initial volume of 0.020 m³ to a final volume of 0.060 m³. For the van der Waals gas,

\[
\left( \frac{\partial U}{\partial V} \right)_{T} = \frac{a}{V_{m}^{2}}. 
\]

Assume that \( a = 0.556 \) Pa m⁶ mol⁻², and that \( b = 64 \times 10^{-6} \) m³ mol⁻¹. Calculate \( q \), \( w \), \( \Delta U \), \( \Delta H \), and \( \Delta S \) for the process.

9) 15.0 grams of steam at 373 K is added to 250.0 g of H₂O (l) at 298 K at a constant pressure of 1 bar. Is the final state of the system steam or liquid water? Calculate \( \Delta S \) for the process.

Chapter 6

1) A sample containing 2.50 mol of an ideal gas at 298 K is expanded from an initial volume of 10.0 L to a final volume of 50.0 L. Calculate \( \Delta G \) and \( \Delta A \) for this process for (a) an isothermal
reversible path and (b) an isothermal expansion against a constant external pressure of 0.750 bar. Explain why $\Delta G$ and $\Delta A$ do or do not differ from one another.

2) Calculate $\Delta G^\circ_{\text{reaction}}$ for the reaction $\text{CO}(g) + \frac{1}{2} \text{O}_2(g) \leftrightarrow \text{CO}_2(g)$ at 298.15 K. Calculate $\Delta G^\circ_{\text{reaction}}$ at 650 K assuming that $\Delta H^\circ_{\text{reaction}}$ is constant in the temperature interval of interest.

3) Consider the equilibrium $\text{NO}_2(g) \leftrightarrow \text{NO}(g) + \frac{1}{2}\text{O}_2(g)$. One mole of $\text{NO}_2(g)$ is placed in a vessel and allowed to come to equilibrium at a total pressure of 1 bar. An analysis of the contents of the vessel gives the following results:

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$P_{\text{NO}}$</th>
<th>$P_{\text{NO}_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>0.872</td>
<td>2.50</td>
</tr>
<tr>
<td>800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Calculate $K_P$ at 700 and 800 K.

b. Calculate $\Delta G^\circ_{\text{reaction}}$ and $\Delta H^\circ_{\text{reaction}}$ for this reaction at 298.15 K. Assume that $\Delta H^\circ_{\text{reaction}}$ is independent of temperature.

c. Calculate $\Delta S^\circ_{\text{reaction}}$ at 700 and 800 K and $\Delta H^\circ_{\text{reaction}}$, assuming that it is independent of temperature.

d. Calculate $\Delta G^\circ_{\text{reaction}}$ for this reaction at 298.15 K.

4) If the reaction $\text{Fe}_2\text{N}(s) + \frac{3}{2}\text{H}_2(g) \leftrightarrow 2\text{Fe}(s) + \text{NH}_3(g)$ comes to equilibrium at a total pressure of 1 bar, analysis of the gas shows that at 700 and 800 K, $\frac{P_{\text{NH}_3}}{P_{\text{H}_2}} = 2.165$ and 1.083, respectively, if only $\text{H}_2(g)$ was initially present in the gas phase and $\text{Fe}_2\text{N}(s)$ was in excess.

a. Calculate $K_P$ at 700 and 800 K.

b. Calculate $\Delta S^\circ_{\text{reaction}}$ at 700 and 800 K and $\Delta H^\circ_{\text{reaction}}$, assuming that it is independent of temperature.

c. Calculate $\Delta G^\circ_{\text{reaction}}$ for this reaction at 298.15 K.

5) The pressure at the center of the earth is probably greater than 3x10^6 atm, and the temperature there is about 4000 °C. What is the change in the Gibbs function of reaction on going from crust to core for a reaction in which $\Delta V_m = 1 \text{ cm}^3 \text{ mol}^{-1}$ and $\Delta S_m = 2.1 \text{ J K}^{-1} \text{ mol}^{-1}$? Does the pressure or temperature effect dominate in determining the change?

6) A sample containing 2.25 moles of He (1 bar, 298 K) is mixed with 3.00 mol of Ne (1 bar, 298 K) and 1.75 mol of Ar (1 bar, 298 K). Calculate $\Delta G^\circ_{\text{mixing}}$ and $\Delta S^\circ_{\text{mixing}}$. 